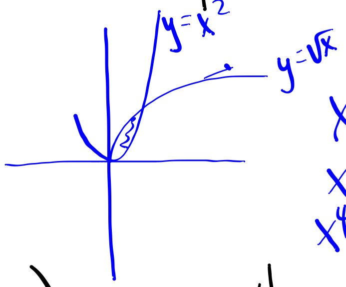
7.1 Area Between 2 Curves

Suppose we want to find the area between $y=x^2$ and $y=\sqrt{x}$.

What would we do?

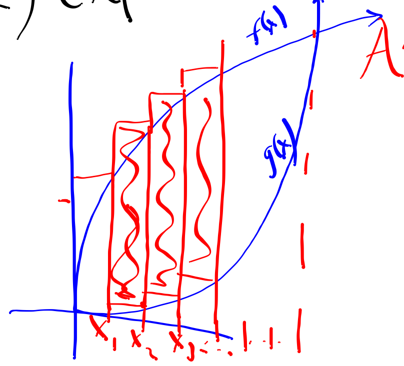
Area Between $y=x^2$ and $y=\sqrt{x}$

1) Find points of intersection,



$$x^2=\sqrt{x}$$
$$x^4=x$$
$$x^4-x=0$$
$$x(x^3-1)=0$$
$$x=0,1$$

2) Express the area as a definite integral



$$A \approx \Delta x_1\left(\sqrt{x_1}-x_1^2\right)+$$
$$\Delta x_2\left(\sqrt{x_2}-x_2^2\right)+$$
$$\Delta x_3\left(\sqrt{x_3}-x_3^2\right)+$$
$$\dots$$

step 2a ... translate into Riemann sum

$$A \approx \sum_{k=1}^{n}\left(\sqrt{x_k}-x_k^2\right)\Delta x_k$$

step 2b ... translate THAT into def. int.

$$\int_0^1\left(\sqrt{x}-x^2\right)dx$$

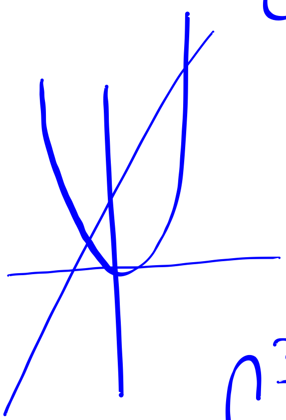
step 3. evaluate the integral

$$\int_0^1 x^{1/2}-x^2\,dx=\left(\frac{2x^{3/2}}{3}-\frac{x^3}{3}\right)\Big|_0^1$$

$$=\left(\frac{2}{3}-\frac{1}{3}\right)-(0)=\frac{1}{3}$$

Find the area between

$y = x+6$ and $y = x^2$



① $x^2 = x+6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0 \Rightarrow x = -2, 3$

$$\int_{-2}^{3} (x+6) - x^2 \, dx =$$

$$= \left(\frac{x^2}{2} + 6x - \frac{x^3}{3}\right)\Bigg|_{-2}^{3} = \left(\frac{9}{2} + 18 - 9\right) - \left(2 - 12 + \frac{8}{3}\right)$$

$$= 19 + \frac{9}{2} - \frac{8}{3}$$

$$= 19 + \frac{27 - 16}{6} = 19 + \frac{11}{6}$$

$$= \frac{125}{6}$$

Barron's 30

$x=t^2-1 \qquad y=t^4-2t^3$

find $\frac{d^2y}{dx^2}$ when $t=1$

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{4t^3-6t^2}{2t}=2t^2-3t$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{\frac{d}{dt}(2t^2-3t)}{\frac{dx}{dt}=2t}$$

$$\frac{d^2y}{dx^2}=\left(\frac{4t-3}{2t}\right)$$

evaluate when $t=1$ .... $\frac{4(1)-3}{2(1)}=\frac{1}{2}$

Barron's 50

$$\lim_{x \to 0} \frac{\sec x - \cos x}{x^2}$$

$$\overset{L'H}{=} \lim_{x \to 0} \frac{(\sec x \tan x) - (-\sin x)}{2x}$$

$$\overset{L'H}{=} \lim_{x \to 0} \frac{(\sec x \tan x)(\tan x) + (\sec x)(\sec^2 x) + \cos x}{2}$$

$$= \frac{2}{2} = 1$$

Barron's 49

The graph.... $x=3+2\sin t$
$y=2\cos t-1$

for $-\pi\le t\le\pi$

is:

$x=3+2\sin t \Rightarrow \sin t=\frac{x-3}{2}$

$y=2\cos t-1 \Rightarrow \cos t=\frac{y+1}{2}$

$\sin^2 t+\cos^2 t=1$

$\left(\frac{x-3}{2}\right)^2+\left(\frac{y+1}{2}\right)^2=1$

$(x-3)^2+(y+1)^2=2^2$

$x^2-6x+9+y^2+2y+1=4$

the entire circle

shape is a CIRCLE
Center $=(3,-1)$
radius is 2

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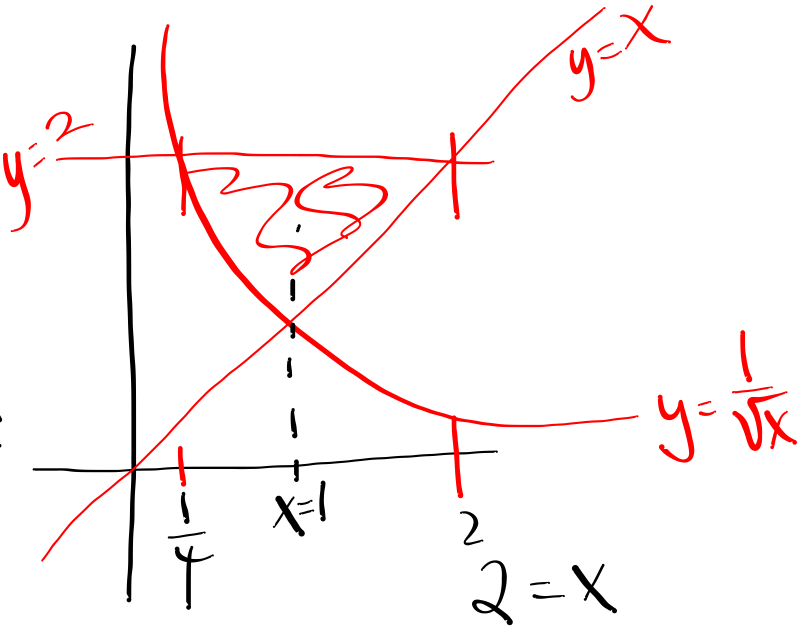
7.1 #3

$y=2=\frac{1}{\sqrt{x}}$

$\sqrt{x}=\frac{1}{2}$

$x=\frac{1}{4}$



$2=x$

$\frac{1}{\sqrt{x}}=x \Rightarrow$

$\frac{1}{x}=x^2$

$1=x^3$

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7.1 #3

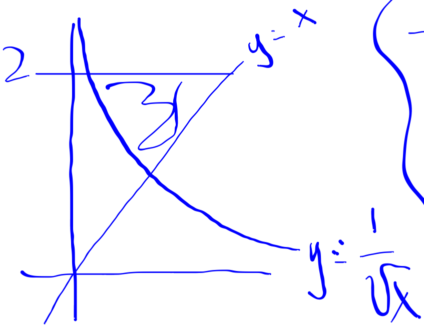
$$A=\int_{\frac{1}{4}}^{1}(2)-\left(\frac{1}{\sqrt{x}}\right)dx$$

left side

$$+\int_{1}^{2}(2)-(x)\,dx$$

To view this problem sort of sideways...

→ replace y with x & x with y

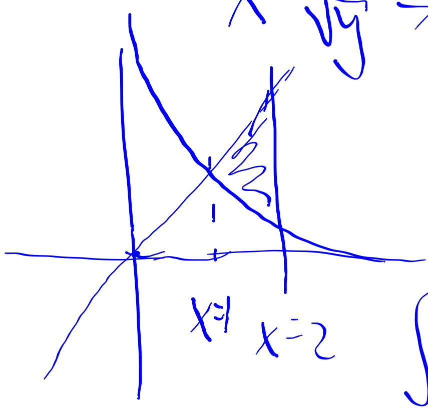


x=2

$y=x$

$x=\frac{1}{\sqrt{y}} \Rightarrow \frac{1}{\sqrt{y}}=x$

$y=\frac{1}{x^2}$



$$A=\int_{1}^{2}(x)-\left(\frac{1}{x^2}\right)dx$$

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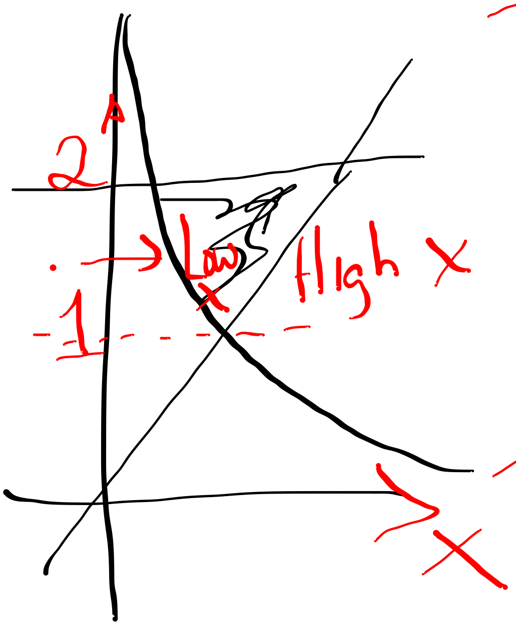
7.1/#3



$y=\frac{1}{\sqrt{x}}$

Solve for x

$\sqrt{x}=\frac{1}{y}$

$x=\frac{1}{y^2}$ low fn on bottom

$A=\int_1^2 \_\_\_\_$

$y=x$

$x=y$ high x fn on top

$$A=\int_1^2 \left(y-\left(\frac{1}{y^2}\right)\right)dy$$

Quiz-Opt # 1

Express the number 25 as the sum of 2 non-negative #s so that product is as large as possible

$x+y=25$

$y=25-x$

$P=x\cdot y$

$P=x(25-x)$

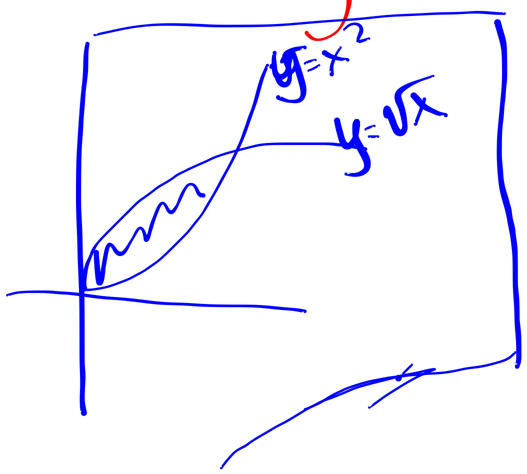
$P=25x-x^2$

$P'=25-2x=0$

$x=12.5$

7.1 Area Between 2 curves

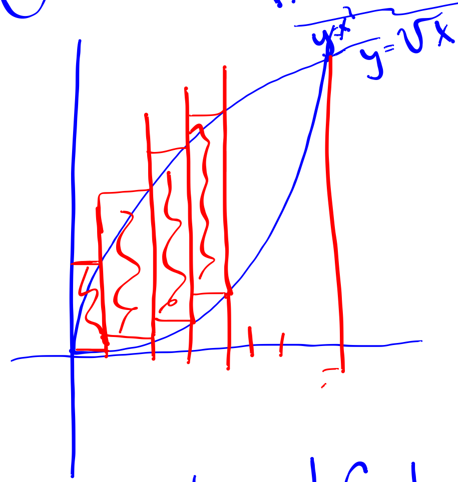
Find the area between $y=x^2$ & $y=\sqrt{x}$.



① find the pts of intersection

$$x^2=\sqrt{x}$$
$$x^4=x$$
$$x^4-x=0$$
$$x(x^3-1)=0 \rightarrow x=0,+1$$

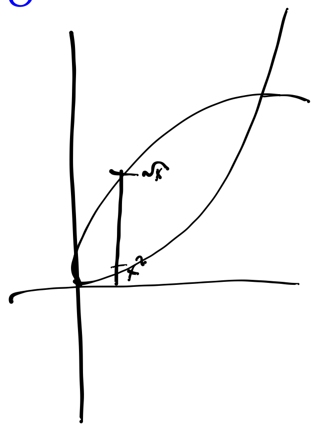
(2) find a Riemann sum



$A \approx \Delta x_1(\sqrt{x_1} - (x_1)^2) +$
$\Delta x_2(\sqrt{x_2} - x_2^2) +$
$\Delta x_3(\sqrt{x_3} - x_3^2) +$
$- - -$

2b) create a definite integral from Riemann sum

$$A \approx \sum_{k=1}^{n} (\sqrt{x_k} - x_k^2)\Delta x_k$$



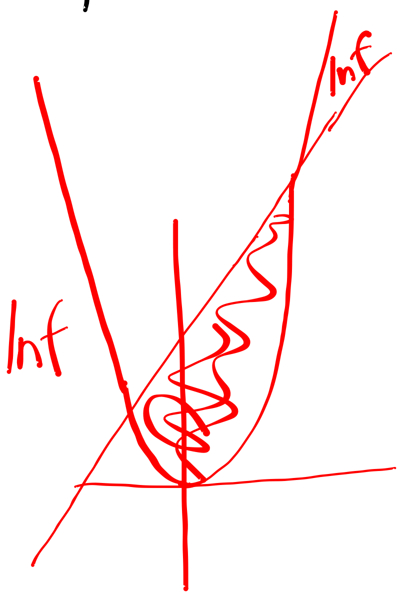
$$\int_0^1 (\sqrt{x} - x^2)\,dx$$

$$= \int_0^1 \sqrt{x}\,dx - \int_0^1 x^2\,dx$$

$$\int_0^1 \sqrt{x} - x^2\,dx = \left(\frac{2x^{3/2}}{3} - \frac{x^3}{3}\right)\Big|_0^1 =$$

$$\left(\frac{2}{3} - \frac{1}{3}\right) - (0 - 0) = \frac{1}{3}$$

Find the area between $y = x+6$ and $y = x^2$.



$x + 6 = x^2$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x = -2, +3$

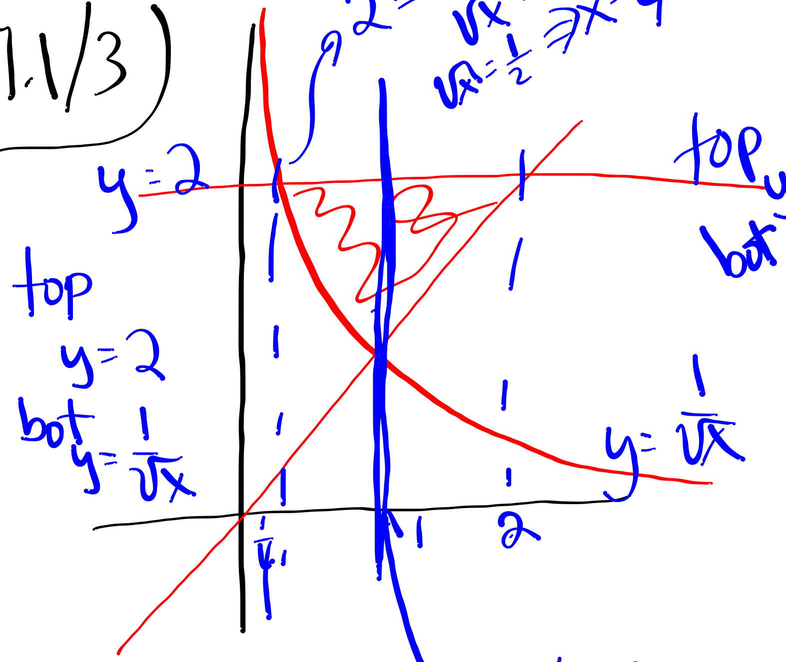
$$A = \int_{-2}^{3} (x+6) - (x^2)\,dx = \left(\frac{x^2}{2} + 6x - \frac{x^3}{3}\right)\Bigg|_{-2}^{3}$$

$$= \left(\frac{9}{2} + 18 - 9\right) - \left(\frac{4}{2} - 12 + \frac{8}{3}\right)$$

$$= 21 + \frac{5}{2} - \frac{8}{3} = 21\frac{1}{6} = \frac{125}{6}$$

7.1/3)

$2 = \frac{1}{\sqrt{x}} \Rightarrow \sqrt{x} = \frac{1}{2} \Rightarrow x = \frac{1}{4}$



top $y=2$
bot $y=x$

$y = x = 2$

Area $= \int_{\frac{1}{4}}^{1} 2 - \frac{1}{\sqrt{x}}\,dx + \int_{1}^{2} 2 - x\,dx$

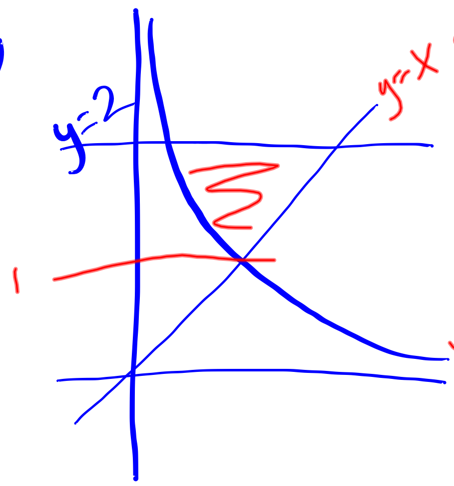
$t^{3/2}$

$\frac{1}{\sqrt{t}} = t^{3}$

$1 = t$

$t = 1$

7.1/3)



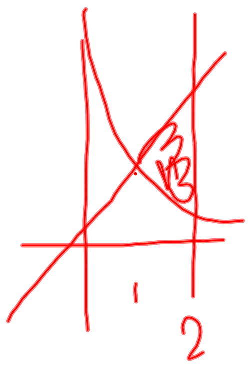
$y=x$ OR l0W...

$x=y$

$y=\frac{1}{\sqrt{x}}$ OR IN OTHER WORDS

$\sqrt{x}=\frac{1}{y}$

$x=\frac{1}{y^2}$



$$A=\int_1^2 y-\frac{1}{y^2}\,dy$$

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